

SMOOTHNESS AND SINGULARITIES OF THE PERFECT FORM COMPACTIFICATION OF \mathcal{A}_g

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ABSTRACT. We prove that the singular locus of the moduli stack $\mathcal{A}_g^{\text{Perf}}$ given by the perfect cone or first Voronoi decomposition has codimension 10 if $g \geq 4$ and we describe all singularities in codimension 10.

1. INTRODUCTION

By the theory of toroidal compactifications admissible $\text{GL}_g(\mathbb{Z})$ -invariant tessellations of the rational closure $S_{\geq 0}^g$ of the space of positive-definite real quadratic forms in dimension g – also known as admissible rational polyhedral decompositions or fans – give rise to so called toroidal compactifications of the moduli space \mathcal{A}_g of principally polarized abelian varieties. This theory was first developed by Ash, Mumford, Rapoport and Tai [AMRT] and has since been used by numerous authors in geometric studies of moduli of abelian varieties. Toroidal compactifications have the property that the boundary is “big”, i.e. has codimension 1. All toroidal compactifications dominate the Satake compactification $\mathcal{A}_g^{\text{Sat}}$ which is set-theoretically given by

$$(1) \quad \mathcal{A}_g^{\text{Sat}} = \mathcal{A}_g \sqcup \mathcal{A}_{g-1} \sqcup \dots \sqcup \mathcal{A}_0.$$

In the literature three different types of decompositions have been studied in detail: the *first Voronoi* or *perfect cone* decomposition, the *second Voronoi* decomposition and the *central cone* decomposition, leading to the corresponding toroidal compactifications $\mathcal{A}_g^{\text{Perf}}$, $\mathcal{A}_g^{\text{Vor}}$ and $\mathcal{A}_g^{\text{Igu}}$ correspondingly, see [Nam] for more details. In recent years the meaning of these various toroidal compactifications has been clarified. The central cone compactification has been identified with the *Igusa* compactification, a (partial) blow-up of the Satake compactification $\mathcal{A}_g^{\text{Sat}}$. The second Voronoi compactification $\mathcal{A}_g^{\text{Vor}}$ has a meaningful interpretation in terms of moduli of abelian varieties, roughly speaking

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its boundary points correspond to degenerate abelian varieties, more precisely to semi-abelic varieties. For details the reader is referred to work by Alexeev [Ale] and Olsson [Ols]. Finally the first Voronoi or perfect cone compactification is well behaved with respect to the minimal model program. Shepherd-Barron [SB] has proved that $\mathcal{A}_g^{\text{Perf}}$ has canonical singularities and that its canonical bundle is ample if $g \geq 12$. Thus $\mathcal{A}_g^{\text{Perf}}$ is a *canonical model* in the sense of the minimal model program if $g \geq 12$.

The construction of toroidal compactifications of the moduli space \mathcal{A}_g very roughly works as follows. One first has to choose an admissible rational polyhedral decomposition of the rational closure $S_{\geq 0}^g$ of the space of positive definite symmetric $g \times g$ matrices. This defines a compactification $\mathcal{A}_g^{\text{tor}}$ of \mathcal{A}_g as a compact analytic space by adding a stratum to each (orbit) of a cone in the chosen decomposition. The codimension of the stratum which is added equals the dimension of the cone. In the cases we have mentioned above the compactification $\mathcal{A}_g^{\text{tor}}$ is in fact a projective variety. Naturally the geometric properties of $\mathcal{A}_g^{\text{tor}}$ depend essentially on the properties of the chosen decomposition.

In this note we are especially interested in the singularities of toroidal compactifications. These singularities arise in two different ways. First of all the symplectic group $\text{Sp}(2g, \mathbb{Z})$ has torsion (different from $\pm \text{id}$). This gives rise to finite quotient singularities in \mathcal{A}_g (unless the torsion element is a reflection, which only happens in genus $g = 2$). Similarly such quotient singularities can arise in the boundary of a toroidal compactification due to non-neatness of $\text{Sp}(2g, \mathbb{Z})$. Such quotient singularities are well behaved from an algebraic-geometric point of view and can for many considerations be neglected: if one replaces the group $\text{Sp}(2g, \mathbb{Z})$ by a principal congruence subgroup $\Gamma(\ell) = \{g \in \text{Sp}(2g, \mathbb{Z}) \mid g \equiv \mathbf{1} \pmod{\ell}\}$, then for $\ell \geq 3$ this group is neat and in particular torsion free. Hence the corresponding level covers $\mathcal{A}_g(\ell)$ respectively $\mathcal{A}_g^{\text{tor}}(\ell)$ will not acquire such singularities. A second type of singularities arises from “bad” behavior of cones σ in the chosen decomposition. Recall that a cone σ whose general element has rank g is called *basic* if the integral generators of its 1-dimensional faces can be completed to a \mathbb{Z} -basis of $\text{Sym}^2(\mathbb{Z}^g)$. It is called *simplicial* if these generators can be completed to a \mathbb{Q} -basis, in other words if the number of generators of the cone equals its dimension. If a cone σ is basic, then its corresponding stratum lies in the non-singular locus of $\mathcal{A}_g^{\text{tor}}(\ell)$ for $\ell \geq 3$, whereas simplicial cones give rise to finite quotient singularities by abelian groups, the group being the quotient of the lattice $\text{Sym}^2(\mathbb{Z}^g)$

by the lattice spanned by the integral generators of the cone. Non-simplicial cones give rise to more general singularities. Note that taking a level cover will not remove singularities which arise from non-basic cones, in other words they are singularities of the corresponding stack. We will refer to these singularities as the *essential* singularities, sometimes these singularities are also called *stacky* singularities.

Knowledge of the singularities of a toroidal compactification $\mathcal{A}_g^{\text{tor}}$ and its level covers is obviously of geometric interest. For $g \leq 3$ the first and second Voronoi decomposition as well as the central cone decomposition all coincide and all cones are basic. Hence there are no essential singularities in genus ≤ 3 . This changes in genus 4. All cones in the second Voronoi decomposition are still basic, but this is no longer the case in the first Voronoi or perfect cone decomposition, which in genus 4 coincides with the central cone decomposition. Here there is one non-basic cone, namely the perfect cone of the root lattice D_4 , which has dimension 10 and 12 rays, hence is neither basic nor simplicial. This defines the unique essential singular point in the 10-dimensional variety $\mathcal{A}_4^{\text{Perf}}$. This also means that for $g \geq 4$ the space $\mathcal{A}_g^{\text{Perf}}$ will always have essential singularities in codimension 10 (or less). There is no a priori reason that the codimension of the essential singular locus of $\mathcal{A}_g^{\text{Perf}}$ could not be less than 10 for $g > 4$. Our aim is to show that this is not the case. We shall also classify the essential singularities in codimension 10 – as it turns out they all come from the root lattice D_4 .

2. THE RESULT

We shall now work with the perfect cone or first Voronoi decomposition and its associated toroidal compactification $\mathcal{A}_g^{\text{Perf}}$. For a vector $v \in \mathbb{Z}^g$ we write $p(v) = vv^t$ for the corresponding rank 1 form and $q_v(x) = (\sum_{i=1}^g x_i v_i)^2$ for the quadratic form. Any cone σ of the perfect cone decomposition is of the form $\sum_{i=1}^N \mathbb{R}_+ p(v_i)$. By $\dim \sigma$ we mean the *dimension* of σ in $\text{Sym}^2(\mathbb{Z}^g)$, which is equal to the codimension of the corresponding stratum in $\mathcal{A}_g^{\text{Perf}}$.

Theorem 1. *For following holds:*

- (i) *Every cone of dimension at most 9 in the perfect cone decomposition is basic. In particular $\mathcal{A}_g^{\text{Perf}}$ has no essential singularities for $g \leq 3$ and the locus of essential singularities has codimension 10 if $g \geq 4$.*
- (ii) *The only stratum of essential singularities in codimension 10 is the one of D_4 .*

Proof. Let us take a cone σ of dimension N of the perfect cone decomposition generated by $p(v_1), \dots, p(v_M)$. Denote by d the dimension of the lattice $L = \mathbb{Z}v_1 + \dots + \mathbb{Z}v_M$. There exists a \mathbb{Z} -basis $\{w_1, \dots, w_g\}$ of \mathbb{Z}^g such that $\{w_1, \dots, w_d\}$ is a \mathbb{Z} -basis of the lattice $(L \otimes \mathbb{R}) \cap \mathbb{Z}^g$.

As a consequence, the property that concerns us, namely that the family $(p(v_i))_{1 \leq i \leq M}$ is extensible to a \mathbb{Z} - or \mathbb{Q} -basis, can be considered by assuming that $d = g$, i.e. that L is a finite index sublattice of \mathbb{Z}^g .

Since $d \leq \dim \sigma$ we have a finite number of cases to consider. An important tool in our work is the enumeration in [E-VGS] of the orbits of cones of the perfect cone decomposition in dimension $g \leq 7$. Combined with the enumeration of 9-dimensional cones of the perfect cone decomposition (Lemma 2 for $g = 8$ and the 31 entries with $s = r = s' = 9$ of Table 2, 3 and 7 of [KMS] for $g = 9$) this give us (i) after checking that all such cones are basic.

Lemma 1 allows us to resolve the cases $g = 8, 9$ and 10 and $\dim \sigma = 10$. Assertion (ii) then follows from the enumeration of all cones for $g \leq 7$ and checking that they are all simplicial except the one of D_4 . \square

Lemma 1. *Let $\sigma = \mathbb{R}_+p(v_1) + \dots + \mathbb{R}_+p(v_M)$ be a cone of the perfect cone decomposition of $\text{Sym}^2(\mathbb{Z}^g)$. Assume that (v_1, \dots, v_M) span \mathbb{R}^g and that $\dim \sigma = g, g + 1$ or $g + 2$. Then σ is simplicial.*

Proof. We can find g linearly independent vectors $(v_i)_{1 \leq i \leq g}$ among the v_i that determine a basis of \mathbb{Q}^g . By using this basis, the vector space spanned by $(p(v_i))_{1 \leq i \leq g}$ can be identified with the space of diagonal $g \times g$ matrices.

Suppose $\dim \sigma = g$. If $v = \sum_{i=1}^g \alpha_i v_i$ with $\alpha_i \neq 0$ for at least two indices $i_1 \neq i_2$, then $p(v)$ is linearly independent of the $p(v_i)$ since the non-diagonal entry (i_1, i_2) is non-zero. So, if $p(v)$ belongs to the vector space spanned by $(p(v_i))_{1 \leq i \leq d}$ then v is a multiple of one v_i and $p(v)$ of $p(v_i)$ as well.

Suppose $\dim \sigma = g + 1$. We can find a vector v in L with $p(v)$ in σ linearly independent of the $p(v_i)$. After suitable scaling we may assume without loss of generality that $v = \sum_{i=1}^r v_i$ with $2 \leq r \leq g$. Suppose now that $w = \sum_{i=1}^g \alpha_i v_i$ with $p(w) \in \sigma$. Then there exist γ and β_i such that $p(w) = \gamma p(v) + \sum_i \beta_i p(v_i)$ and we have the equation

$$\left(\sum_{i=1}^g \alpha_i x_i \right)^2 = \gamma \left(\sum_{i=1}^r x_i \right)^2 + \sum_{j=1}^g \beta_j x_j^2$$

for the corresponding quadratic forms. From this it follows that we cannot have $\alpha_i \alpha_j \neq 0$ for $i \geq r + 1$ and $i \neq j$. This implies that either

$\alpha_i \neq 0$ only for one $i_0 \geq r+1$ or $\alpha_i = 0$ for $i \geq r+1$. The first case leads to $w = \alpha_{i_0} v_{i_0}$ and $p(w)$ a multiple of $p(v_{i_0})$. The second case leads us to $\alpha_i \alpha_j = \gamma$ for $i, j \leq r$ and $i \neq j$, so in particular $\alpha_i \neq 0$ for $i \leq r$. Assume now $r \geq 3$. At least two such α_i are of the same sign and so $\gamma > 0$. So, all non-zero α_i are of the same sign which we can assume positive. For all subsets $S = \{i_1, i_2, i_3\}$ of $\{1, \dots, r\}$ the equations $\alpha_{i_1} \alpha_{i_2} = \alpha_{i_1} \alpha_{i_3} = \alpha_{i_2} \alpha_{i_3} = \gamma$ have the unique solution $\alpha_{i_1} = \alpha_{i_2} = \alpha_{i_3} = \sqrt{\gamma}$, so w is actually collinear to v . In the case $r = 2$, we have $\beta_i = 0$ for $i \geq 3$ and we can actually restrict the problem to the case of dimension $n = 2$. In that case the tessellation by perfect domains is actually the classical Modulfur and all its cones are simplicial (see, for example, [Sch]).

Suppose $\dim \sigma = g+2$. By the same argument as before we can find vectors of the form $v = \sum_{i=1}^r v_i$ and $v' = \sum_{i=1}^g \lambda_i v_i$ such that the set $\{p(v_1), \dots, p(v_g), p(v), p(v')\}$ is a basis of the real vector space $\text{Vect } \sigma$ in $\text{Sym}^2(\mathbb{Z}^g)$ spanned by σ . Let us write $w = \sum_{i=1}^g \mu_i v_i$ and assume that $p(w) \in \text{Vect } \sigma$. So, there exist α_i, β, γ such that in terms of quadratic forms we have

$$\sum_{i=1}^g \alpha_i x_i^2 + \beta \left(\sum_{i=1}^r x_i \right)^2 + \gamma \left(\sum_{i=1}^g \lambda_i x_i \right)^2 = \left(\sum_{i=1}^g \mu_i x_i \right)^2.$$

By the preceding argument we can assume $\beta \neq 0$ and $\gamma \neq 0$. Assume $r < g$. Then $\gamma \lambda_i \lambda_g = \mu_i \mu_g$ for $i < g$. If $\lambda_g = 0$ then $\mu_i \mu_g = 0$. If $\mu_g \neq 0$ then $\mu_i = 0$ and w is collinear to v_g . If $\mu_g = 0$ then we have necessarily $\alpha_g = 0$ and the problem is reduced to a lower dimensional one. So, $\lambda_g \neq 0$ and by the same argument $\mu_g \neq 0$. So $\lambda_i = \mu_i \kappa$ with $\kappa = \mu_g / (\gamma \lambda_g)$ for $i < g$. Hence the quadratic forms $p(v')$ and $p(w)$ are multiples of each other on the restriction to the hyperplane $x_g = 0$ and the preceding argument for $\dim \sigma = g+1$ allows us to conclude in that case.

So $r = g$ and we get for all $i \neq j$ the equality $\beta + \gamma \lambda_i \lambda_j = \mu_i \mu_j$. We can assume that all λ_i and μ_i are non-zero since otherwise we can do permutation between v, v' and w and reduce ourselves to the preceding case. We can also assume that $g \geq 4$ since the decompositions for $g = 2, 3$ are known to be simplicial. Without loss of generality we can assume $\mu_g = 1$. So, we get $\mu_i = \beta + \gamma \lambda_i \lambda_g$ and for $1 \leq i < j < g$:

$$\begin{aligned}
 0 &= \mu_i \mu_j - \beta - \gamma \lambda_i \lambda_j \\
 &= (\beta + \gamma \lambda_i \lambda_g)(\beta + \gamma \lambda_j \lambda_g) - \beta - \gamma \lambda_i \lambda_j \\
 &= \beta^2 - \beta + \beta \gamma \lambda_g (\lambda_i + \lambda_j) + \gamma (\gamma \lambda_g^2 - 1) \lambda_i \lambda_j \\
 &= a + b(\lambda_i + \lambda_j) + c \lambda_i \lambda_j.
 \end{aligned}$$

If $ac \neq b^2$ then there exists a fractional function $\phi : \mathbb{P}^1(\mathbb{R}) \rightarrow \mathbb{P}^1(\mathbb{R})$ such that $\lambda_i = \phi(\lambda_j)$ and $\phi \circ \phi = \text{Id}$. So, there are one or two possible values for λ_i and if it is two then for any $1 \leq i < j < g$ if λ_i is one then λ_j is the other. The restriction $g \geq 4$ implies that λ_i takes only one value in this case. If $ac = b^2$ then we have $c \neq 0$ and $(\lambda_i + b/c)(\lambda_j + b/c) = 0$. So, no two λ_i can be different from $-b/c$. If at least one of them, say λ_{i_0} is different from $-b/c$ and λ_g as well then by selecting another index than g we reach a contradiction. So, all λ_i , except possibly λ_g , are equal. If they are all equal then v, v' and w are collinear. If they are not, then we obtain $\alpha_i = 0$ for $i < g$ and so the problem is restricted to the space $\sum_{i=1}^{g-1} x_i = 0$ and so is two-dimensional. But we have already seen that all cones for $g = 2$ are simplicial. \square

Lemma 2. *There are 106 orbits of 9-dimensional cones in the perfect cone decomposition of the perfect cone decomposition in dimension 8.*

Proof. By Lemma 1 such cones are simplicial, i.e. of the form $\mathbb{R}_+p(v_1) + \dots + \mathbb{R}_+p(v_9)$, and without loss of generality we can assume that (v_1, \dots, v_8) are independent. The 8-dimensional cones of the perfect cone decomposition in dimension 8 are classified in [Ma] (see entries satisfying $n = s = r = s'$ in Tableau 11.1 there) and there are 13 such orbits. So, we can assume that (v_1, \dots, v_8) is a representative of one of those 13 orbits. Let L be the sublattice spanned by these vectors. The index $i(L)$ of the sublattice L of \mathbb{Z}^8 that they define satisfies $1 \leq i(L) \leq 5$. We can assume that (v_1, \dots, v_8) is of maximal index among all nine 8-subsets of (v_1, \dots, v_9) . For any $1 \leq j \leq 8$ we can define the lattice

$$L_j = \mathbb{Z}v_1 + \dots + \mathbb{Z}v_{j-1} + \mathbb{Z}v_{j+1} + \dots + \mathbb{Z}v_8 + \mathbb{Z}v_9.$$

If L_j is full dimensional then the index $i(L_j)$ of L_j is at most $i(L)$ and so

$$|\det(v_1, \dots, v_{j-1}, v_{j+1}, \dots, v_8)| \in \{1, \dots, i(L)\}.$$

If L_j is not full-dimensional then the determinant is 0. So, there are $2i(L) + 1$ possibilities for the determinant and since there are 8 of them we have a priori $(1 + 2i(L))^8$ possibilities. By using the exterior product, we can rewrite the equations as $-5 \leq \langle w_j, v_9 \rangle \leq 5$ with w_j an integral vector. This defines a parallelohedron and we use the program `zsolve` from 4ti2¹ for enumerating its integral points [HM]. Then we use the automorphism group of the configuration (v_1, \dots, v_8) for identifying the orbits. For each such orbit we first check that each of

¹Web page <http://www.4ti2.de/>

the 8-subsets corresponds to one of the 13 orbits or is linearly dependent and if so then we test if the candidate (v_1, \dots, v_9) is realizable by using Algorithm 1 of [KMS]. We then remove isomorphic pairs among the cones thus obtained and get the 106 orbits. \square

Remark. We would like to conclude the paper with some comments on the algorithm which was employed in the enumeration. An essential step is of course Lemma 1 which makes the enumeration feasible in the first place. Another point is the following: while the search space is of size comparable to $(1 + 2i(L))^8$, the determinant of the matrix (w_j) is $i(L)^7$, which means that the number of integer solutions is actually reasonable and this allow `zsolve` to find all solutions. So, the condition that $\{v_1, \dots, v_8\}$ is of maximal index is the key to the enumeration. The total running time was 8 hours and is dominated by two aspects. One is the test for equivalence and computing automorphism groups of system of shortest vectors. The other is testing realizability of families of vectors.

For a given shortest family $\mathcal{V} = (\pm v_1, \dots, \pm v_8)$ the group G_2 of rational automorphism is of size $2^8 8!$. In order to obtain the integral automorphisms, we first determine a subgroup G_1 formed by transpositions (i, j) and sign changes $(v_i \mapsto -v_i)$ which induce integral automorphisms. The full symmetry group is then obtained by iterating over double cosets $G_1 h G_1$ and keeping the ones that preserve \mathcal{V} .

For realizability tests, we use eigenvectors in order to get vectors of negative norms when the solution is not positive definite. The required number of iterations can be very large and in order to have reasonable time runs, we remove vectors which have large norm with respect to the linear programming solution. But if those vectors show up again in the process, we have to keep them till the end since otherwise we have the well known phenomenon of zigzagging.

The method used for proving Lemma 2 could possibly be used in dimension 9 but the complexity would be much larger. Extending the work in [Sch] to dimension 10 is likely to be harder still. A simpler task would be to compute the cones of the perfect cone decomposition of dimension $8 + k$ for k small and $g = 8$. Lemma 2 gives the list for $k = 1$ and this is a much smaller number than that for perfect forms (10916 for $g = 8$ [DSV]). Hence, this could be useful for group cohomology computations since the computation would be more manageable.

Unfortunately, we are not aware of a general method for proving basicness without full enumeration.

There are examples of cones which are simplicial but not basic. One such example is given by the shortest vectors of the dual root lattice E_7^* (the index of the corresponding sublattice of $\text{Sym}^2(\mathbb{Z}^7)$ is 384).

Question. If we take $v_1 = (1, 1)$ and $v_2 = (1, -1)$ then we get the formula

$$\frac{1}{2}(q_{v_1}(x) + q_{v_2}(x)) = x_1^2 + x_2^2$$

showing that $\{q_{v_1}, q_{v_2}\}$ cannot be extended to a \mathbb{Z} -basis of $\text{Sym}^2(\mathbb{Z}^2)$. Of course $\{\pm v_1, \pm v_2\}$ cannot be realized as the set of shortest vector of a 2-dimensional lattice. This raises the following question. Assume that $\{v_1, \dots, v_g\}$ are independent vectors with $v_i \in \mathbb{Z}^g$, such that $\{p(v_1), \dots, p(v_g)\}$ is a perfect cone. Is it true that this cone is basic? The answer is positive for $g \leq 9$ as we have shown here.

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